5.3 Modeling of the Annual Cycle of Soil Moisture[1]

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with an Appendix

Evapotranspiration Climatonomy of Drainless Areas

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1 Introduction

Primary productivity depends strongly on seasonal variations of soil moisture and evaporation. It is also known that the amount of soil moisture and its rate of exchange, both indispensable factors of any plant-climate model, are difficult to assess. For example, Lemon *et al.* (1971) mention specifically that soil moisture variations and evaporation processes are not very successfully simulated by the so-called "SPAM" model with which he describes the "Sun's Work in a Cornfield." This steady state model is an example of the microstructure approach with emphasis on the complexities of a specific plant canopy. On the other end of the spectrum are global-scale models as exemplified by Lieth and Box (1972), who simulate world-wide relationships between primary productivity and annual averages of evapotranspiration in the "Thornthwaite Memorial Model."

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In the following I would like to describe an application of a model developed by H. Lettau (1969, see Appendix) which is based on the annual variations of energy and mass supply as input and generates the constituents of the hydrological cycle as output. The basic water balance equation is solved by rigorous numerical integration following parameterization of the process of evapotranspiration.

2 Community Types and Evapotranspiration Climatonomy in a Steppe Environment

We know that the moisture factor is most crucial in semiarid climates. Figure 1 shows a set of data from the semiarid region of eastern Washington as reported by Daubenmire (1972) in his discussion of annual cycles of soil moisture and temperature as related to grass development in a steppe environment. The actual soil moisture distributions in eight climax steppe communities demonstrate significant differences in the soil moisture profiles of these plant communities, although the abiotic input, i.e., precipitation and global radiation, was presumably the same. Daubenmire concludes that the plant communities themselves influence the soil moisture status—in other words, some communities conserve water better than others—but he cautions that these conclusions should be validated by observations in different years and in various stands.

We can try to compare Daubenmire's observations with an evapotranspiration climatonomy model to see what levels of soil moisture can be predicted if macroclimatic factors are the same but with the plant communities having different hydrological characteristics.

Briefly, the model makes use of two parameters. The first is called evaporivity, and denotes that fraction of precipitation which is returned to the air via evaporation so quickly that it cannot contribute to soil moisture storage changes. That portion of the precipitation which is actually stored underground may be spent either quickly or slowly, and this variable decay rate is measured by the second parameter called residence time. (For further details and mathematical formulations, see the Appendix.)

Figure 2 shows the annual variation of precipitation and global radiation for Spokane, Washington, from August 1962 to August 1963, which was, according to Daubenmire, not an unusual year. Because there is no runoff in the area of Daubenmire's observations, the annual total evaporation must be equal to the annual total precipitation, which was 408 mm. In spite of significant differences in evaporivity and residence time, the calculated monthly evapotranspiration values show little contrast, less than 10 mm per month. But the soil moisture regimes are significantly different (by a factor of 2) as shown in Fig. 3. In comparison with Daubenmire's measurements, remember that the model calculations produce soil moisture totals which correspond to vertically integrated soil moisture per unit volume. Nevertheless, a certain threshold value of m will correspond to the wilting coefficient and another will indicate the approach to field capacity (compare Equations 1 and 4 in the Appendix). For Daubenmire's cases these threshold values are about 55 and 110 mm of water column, respectively. Evaporivity and residence time affect the seasonal variations of the soil moisture regime very distinctly. The soil with high evaporivity and low residence time can be compared with a frivolous spender who dispenses immediately of most of what he receives and disposes of the rest within a short time. The differences in the hydrological

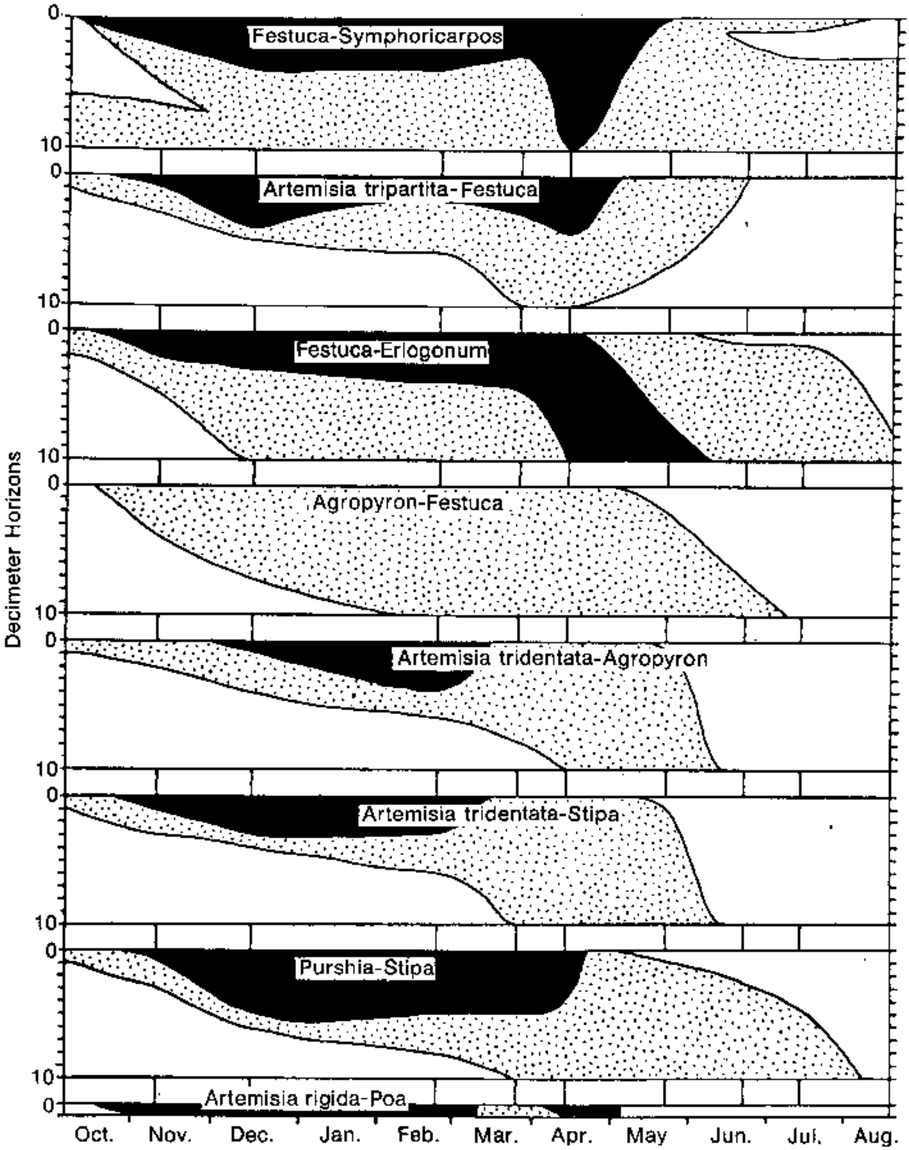

Fig. 1. Soil moisture status in eight climax steppe communities in eastern Washington from October 6, 1962 to August 17, 1963. Limits of decimeter horizons shown on ordinates. Black areas indicate water content at or in excess of the field capacity; stippled areas indicate water content between field capacity and wilting coefficient; unshaded areas lacked growth water. Vertical lines between horizontal panels show actual dates of sampling. (After Daubenmire, 1972.)

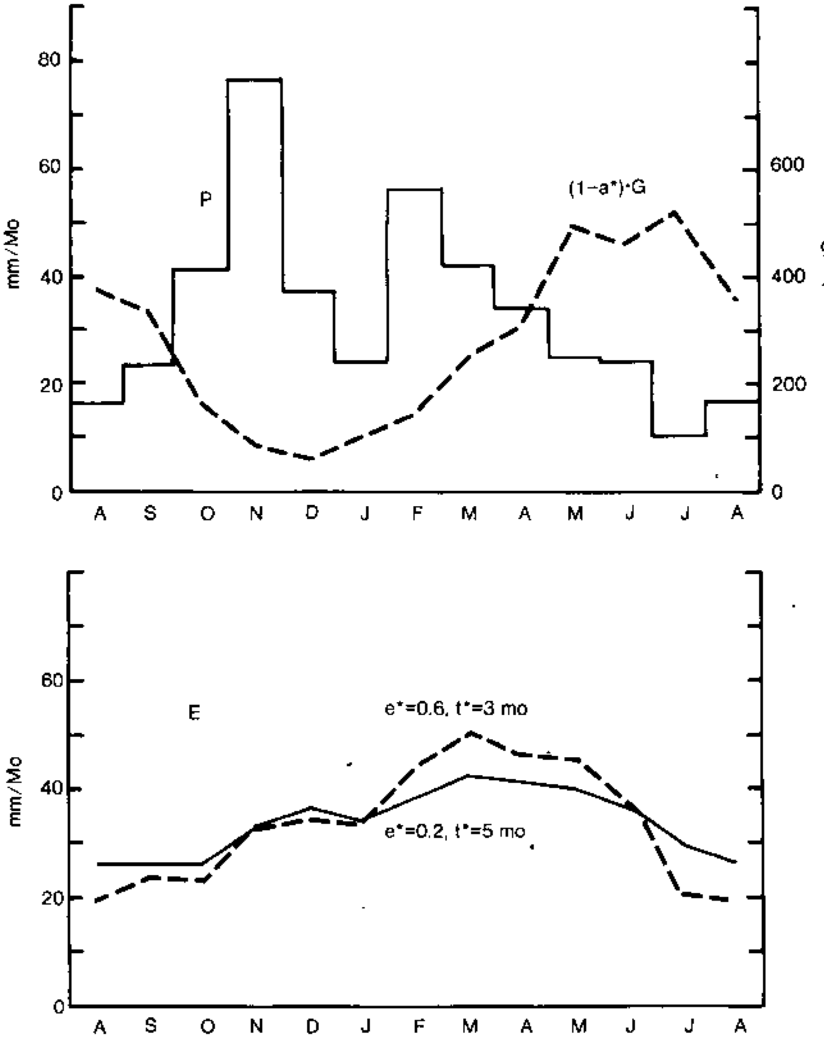

Fig. 2. *Upper*: Observed monthly means of precipitation P and global radiation $(1\text{-}a^*) \cdot G$ for Spokane, Washington, in 1962–1963. *Lower*: Calculated evapotranspiration E (mm/mo) for two sets of parameter values of evaporivity e^* and residence time t^*.

cycles documented by the three calculated curves are significant in terms of threshold values for wilting point and water saturation, and we conclude that vegetation differences could significantly control the hydrological cycle.

3 Computer Simulation of Seasonal Soil Water Changes

Daubenmire emphasized that his measurements should be repeated in different stands and under different climatic conditions. The advantage of a quantitative

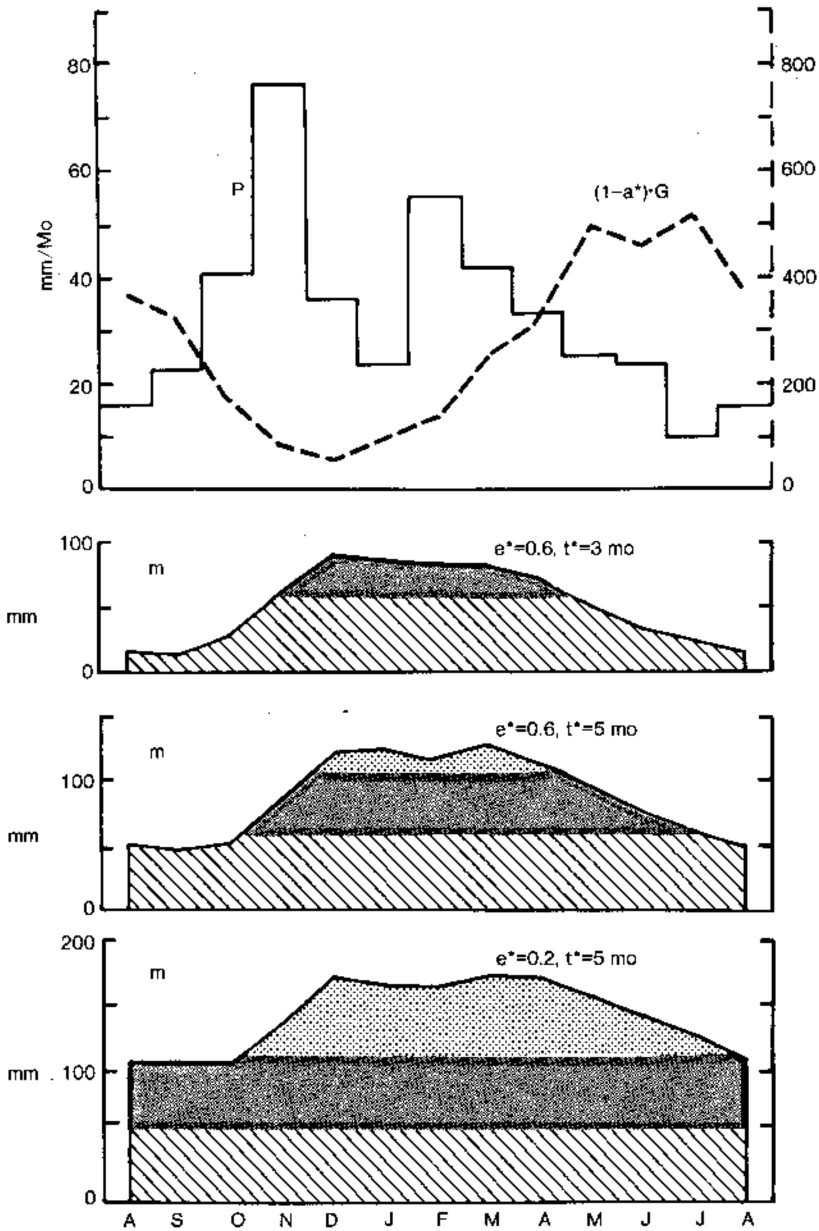

Fig. 3 *Upper*: Observed monthly means of precipitation P and global radiation $(1\text{-}a^*)\cdot G$ for Spokane, Washington, as in Fig. 2. *Lower*: Calculated monthly means of exchangeable soil moisture m for three sets of parameter values of evaporivity e^* and residence time t^*. The horizontal lines at 55 and 110 mm soil moisture indicate approximate threshold values for wilting and field capacity, respectively, as suggested by Daubenmire in Fig. 1.

model is that such effects can be simulated by the computer. Figure 4 illustrates such an application. For a sequence of several years the annual insolation curve is assumed to be equal, whereas the monthly precipitation values were doubled for one year and halved for another year with "normal" years before, in between, and after. The soil moisture curves were calculated with the same set of parameters as before; evidently water is carried forward from "fat" to "lean" years if the soil parameters favor water preservation. Model calculations of this type could serve to simulate conditions in different plant communities and could perhaps be related to variations in productivity. Further research is necessary to clarify problems which may be due to adaptation and feedback processes.

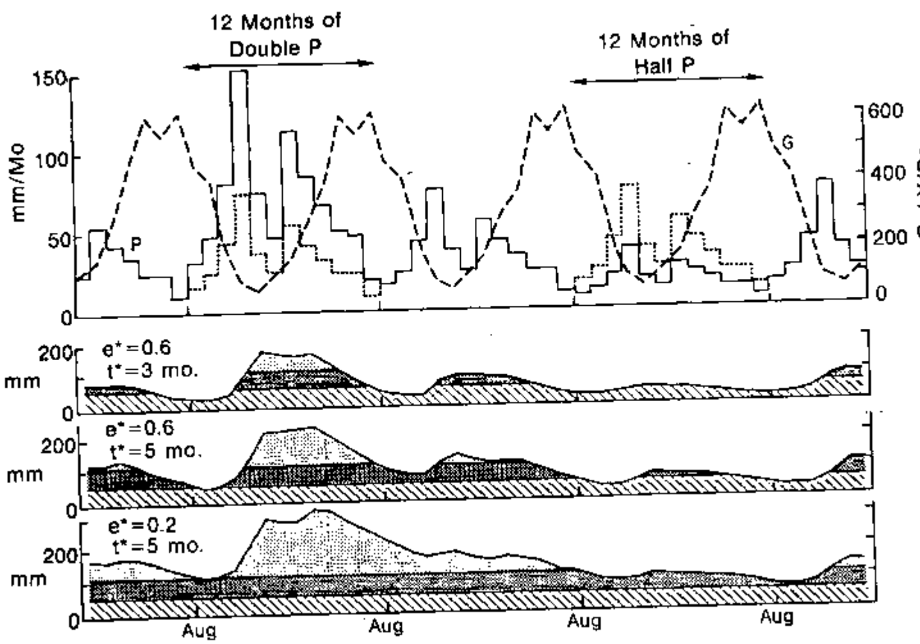

Fig. 4. Results of a model experiment showing the effects on soil moisture variations for a sequence of four years for three sets of parameter values (as in Fig. 3). The seasonal variation of insolation is assumed to be the same each year as it was in 1962–1963, whereas the precipitation of August 1962 to August 1963 is doubled during one 12-month period and halved during another 12-month period.

Nothing has been said about soil and air temperature. Climatonomical models of the complete energy budget at the soil-air interface are available which can predict soil and air temperature by parameterizing the processes of radiation, conduction, and convection. Considering that latent heat is such an important contributor to the total energy budget, and that apparently modeling of soil moisture cycles presents problems in presently available plant-climate modeling, it seemed justified to concentrate explicitly on the hydrological cycle.

4 Appendix: Evapotranspiration Climatonomy of Drainless Areas

Evapotranspiration climatonomy (Lettau, 1969) has recently been extended by the consideration of seasonal variations of the parameters (Lettau and Baradas, 1973) which are used to express the processes of evaporation and runoff. Presently, the model is being extended to predict water level changes of inland lakes and reservoirs. However, the principle of the concept and the mathematical characteristics of the model algorithm are readily demonstrated by an application to the drainless plains of a semiarid region. A steppe environment as discussed in the preceding text offers an instructive example for utilization of climatonomy.

To present the theoretical background for K. Lettau's discussion of abiotic forcing in the Spokane region, and to establish a basis for future applications, a brief summary of the algorithm appears to be in order. We utilize neither regression equations nor other statistical formulas for predicting evapotranspiration. We start out with the basic water balance equation of a soil column and solve it by rigorous numerical integration, following a suitable physical parameterization of the process of evapotranspiration.

For a drainless area the water balance equation has only three terms, which represent input, process, and balancing response functions:

$$P - E = dm/dt \tag{1}$$

where t = time = independent variable (with differentials dt normally interchanged by finite increments Δt, for example, 1 Mo = one month); m = exchangeable soil moisture (mm H_2O) = dependent variable; P = precipitation (including, if applicable, irrigation) = forcing function of mass input (mm/Mo); and E = evapotranspiration = process function (mm/Mo). The variable $m(t)$ is the response function (a function of time) because the mass input P must be prescribed as a time series. The derivative dm/dt (or, alternately, the difference quotient $\Delta m/\Delta t$) represents the storage term. Moisture can be stored either on the surface (snow pack, rain puddles, etc.) or in pores and other cavities within the soil, or by vegetation (within roots, stem, fruits, or leaves, etc.). Soil moisture measurements from layer to layer refer to a volume or weight basis, and such samplings must be vertically integrated (between the surface and the depth of vanishing changes over all seasons) to yield the value of m (expressed as millimeters of H_2O). Namely, exchangeable soil moisture m is the total water column involved in any climatic cycle, including multiannual climatic trends, at a given locality.

The main characteristic of climatonomy is a parameterization of the process function to the effect that the moisture balance Equation 1 can be rigorously integrated. To achieve this, we separate the process E into two additive terms:

$$E = E' + E'' \tag{2}$$

assuming that during any time increment Δt, the first term E' is directly proportional to input of mass (P) as well as to absorption of solar energy, while the remainder term $E'' = E - E'$ varies in proportion to the variable soil moisture m. The two parameters necessary to express these two proportionalities are $e^* =$ evaporivity (dimensionless) and $t^* =$ soil moisture residence time (expressed in units of Δt). Equations of definition are

$$E' = e^* P(1-a^*)\, G/\overline{(1-a^*)G} \tag{3}$$

$$E'' = m/t^* \tag{4}$$

where $G =$ global radiation $=$ forcing function of solar energy interception; and $a^* =$ surface albedo so that $(1-a^*)G =$ absorbed solar energy. Both quantities a^* and G must be representative of the same locality for which P is prescribed. In Equation 3, and in the following formulas the overbar denotes an annual average, so that in Equation 3 the units for G are canceling out. Normally, langleys per day will be used, where one langley (1 ly) is defined as one gram calorie per square centimeter.

In view of Equations 3 and 4 it is convenient to define the reduced input function P':

$$P' = P - E' = P \cdot [1 - e^* \cdot (1-a^*)G/\overline{(1-a^*)G}] \tag{5}$$

and, furthermore, a dimensionless time τ, using t^* as a scaling unit:

$$d\tau = \frac{dt}{t^*}; \quad \text{or,} \quad \tau = \int_0^t \frac{dt}{t^*} \tag{6}$$

With the aid of Equations 5 and 6 we obtain from equation 1 the transformed balance equation

$$dm/d\tau = -m + t^* P' \tag{7}$$

Rigorous integration of Equation 7 yields the response function

$$m = e^{-\tau}(m_0 + \int_0^t P'\, e^{+\tau} dt) \tag{8}$$

where m_0 is the initial value of m at $t = 0$. Once the time series of m has been calculated on the basis of the given inputs P and G, together with prescribed parameters e^* and t^*, we use Equations 4 and 2 to determine the time series of E'' and finally E.

Equations 5 through 8 serve to clarify the physical meaning of the two parameters e^* and t^*. Equation 5 shows that evaporivity e^* determines for any time increment Δt the fraction of P that is returned "quasi-immediately" to the atmosphere due to energy absorption during the same period Δt. Consequently, only the reduced input $P' = P - E'$ is responsible for actual storage changes from one Δt to the next Δt. In this respect the parameter e^* resembles a^*, since albedo also determines the fraction of an intercepted flux (in this case of G) that is returned immediately to the atmosphere. Like albedo, evaporivity can only be determined empirically, from flux measurements. Tentative analyses thus far indicate that e^* for natural watersheds may vary between 0.2 and 0.8. Factors which produce relatively high evaporivity are, for example, rains of the shower type which leave puddles to be acted upon by intermittent sunshine; or, a soil structure that prevents rain water from seeping quickly down to the lower subsoil layers, etc.

The physical meaning of the parameter t^* is most clearly illustrated if we consider Equation 8 for the special case of a dry spell, during which $P = P' = 0$ for a sequence of Δt periods, after an initial m_0 value had been established by earlier rains. In this case the m function decays as $m_0 \cdot e^{-\tau}$ which would be a truly exponential decay if, additionally, t^* would be a constant. In any case the t^* parameter is related to the "half-time scale" similar to that of other natural decay processes. Results obtained thus far (Lettau and Baradas, 1973) indicate that t^* has averages of two to four months and may show a seasonal range between 1.5 and about 10 months. Environmental factors which produce relatively large t^* values are, for example, dense tall vegetation, a high porosity of the soil, or a frozen state of ground water.

In a characteristic departure from conventional models [including the methods by Thornthwaite, Penman, Budyko, or others, as reviewed in standard textbooks, for example, Sellers (1965)], the water balance equation is here solved by integration rather than by algebraic accounting procedures, and climatonomy directly produces the annual average and the seasonal variation of exchangeable soil moisture as well as the "natural" value of m_0 for the month at which the study of the annual cycle is begun. A flexible computer program has been developed by P. Guetter at the University of Wisconsin which can be used to perform the integrations and process calculations for sequences of several years, once the variable time series for forcing functions and parameter values are prescribed.

The program is also flexible with respect to the possible consideration of runoff and run-in processes. These could be important even in a steppe region, which is drainless on the large scale, but may show horizontal movement of moisture from convex to concave terrain structures on a relatively small scale. It is important that the product of $P \cdot G$ (inputs of mass and energy) determines the magnitude of P' in Equation 5 to the effect that the average and the annual variations of m depend significantly on the covariance (for annual cycles: the phase difference) of the inputs P and G. Computer experiments can be performed which demonstrate that the soil moisture regime for a climate with winter rains (Mediterranean type) differs significantly from that with summer rains (monsoon type) even though averages of P and G may be nearly identical. Once the parameters of a region have been established, the model lends itself directly to the prediction of soil moisture variations during periods of either excessive rains or drought. The advantage of the model is that water utilization is limited by the natural supply of mass and energy, and that somewhat artificial concepts of "potential evapotranspiration" or "evaporation under conditions where water is not a limiting factor" are avoided. The major problem areas remaining are the determination of parameter values from primary information and the consideration of possible feedback between soil moisture levels and either parameters or process.

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